

Inflationary cosmology and thermodynamics

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Abstract.

We present a simple and thermodynamically consistent cosmology with a phenomenological model of quantum creation of radiation due to vacuum decay. Thermodynamics and Einstein's equations lead to an equation in which H is determined by the particle number N . The model is completed by specifying the particle creation rate $\Gamma = \dot{N}/N$, which leads to a second-order evolution equation for H . We propose a simple Γ that is naturally defined and that conforms to the thermodynamical conditions: (a) the entropy production rate starts at a maximum; (b) the initial vacuum (for radiation) is a non-singular regular vacuum; and (c) the creation rate is initially higher than the expansion rate H , but then falls below H . The evolution equation for H then has a remarkably simple exact solution, in which a non-adiabatic inflationary era exits smoothly to the radiation era, without a reheating transition. For this solution, we give exact expressions for the cosmic scale factor, energy density of radiation and vacuum, temperature, entropy and super-horizon scalar perturbations.

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1. Introduction

The creation of matter and entropy from vacuum has been studied via quantum field theory in curved spacetime (see for example [1, 2]). Most cosmological models exhibit a singularity which presents difficulties for interpreting quantum effects, because all macroscopic parameters of created particles are infinite there. This leads to the problem of the initial vacuum. A regular vacuum for a species of created particles can be defined in simple terms as a state where all mean values describing the particles, such as energy density, number density, entropy etc., are zero. But this simple condition is not achieved in many scenarios, so that either one has to postulate an initial state beyond the

singularity, or to assume that there was a nonzero number of particles at the initial vacuum.

One attempt to overcome these problems is via incorporating the effect of particle creation into Einstein's field equations. For example, in the papers of the Brussels group [3], the quantum effect of particle creation is considered in the context of the thermodynamics of open systems, where it is interpreted as an additional negative pressure, which emerges from a re-interpretation of the energy-momentum tensor. This effect is irreversible in the sense that spacetime can produce matter, leading to growth of entropy, while the reverse process is thermodynamically forbidden.

The main difference with our present paper is that in [3] the law of massive particle creation, i.e. the mechanism of energy flow from the gravitational field to matter, leads to a non-zero number of particles at the beginning of expansion, described as a fluctuation of the regular vacuum. These results were recently generalized in a covariant form in [4]. Our approach differs from that of [3, 4] in that we do not modify the field equations. Instead, we interpret the source of created particles as a decaying vacuum, described phenomenologically by a time-dependent cosmological 'constant' $\Lambda(t)$.

A number of decaying vacuum models has appeared in the literature (see [5, 6] and references cited there). Inflationary models with fixed cosmological constant and cold dark matter have been successful in accounting for the microwave background and large-scale structure observations, while also solving the age problem. However, these models are challenged by the reduced upper limits on Λ arising from the Supernova Cosmology Project, and also by the long-standing problem of reconciling the very large early-universe vacuum energy density with the very low late-universe limits [6]. One resolution of these problems is a decaying Λ . In common with [5], we attempt to provide some clear and consistent physical motivation for the particular form of vacuum decay, rather than an ad hoc prescription. In ad hoc prescriptions, the functional form of $\Lambda(t)$ or $\Lambda(a)$ (where a is the scale factor) is effectively assumed a priori. Often power-law forms for Λ are assumed (see, for example, [7] and references cited there). Exponential decay laws have also been assumed [8]. Typically, the solutions arising from ad hoc prescriptions for Λ are rather complicated, and moreover, it is often difficult to provide a consistent simple interpretation of the features of particle creation, entropy and thermodynamics.

In contrast to many other models, we propose a simple, exact and thermodynamically consistent cosmological history. The latter originates from *a regular initial vacuum with a maximal initial entropy production rate*. Together with a naturally defined creation rate, this leads to a simple expansion law and thermodynamic properties, and to a definite estimate for the total entropy in the universe. The very existence of an initial maximal entropy production rate reflects a subtle interplay between the conservation equation, the second law of thermodynamics and Einstein's equations, which is at the

heart of our model.

Non-adiabatic inflationary models differ from the standard models (see for example [9]), in that: (a) radiation is created continuously during inflation, rather than during reheating; (b) the continuous vacuum decay itself initiates a smooth exit from inflation to the radiation era; (c) entropy and heat production take place continuously, without the need for reheating. In the standard approach, the scalar field drives adiabatic (i.e., isentropic) inflation, followed by a non-equilibrium reheating era when the field decays into radiation and inflation is brought to an end. The potential of the field is then the key ingredient. In the alternative approach, the key ingredient is essentially the model of vacuum decay. In contrast to the ad hoc models that assume functional forms for Λ , and as an alternative to the thermodynamically motivated model of [5], we arrive at a phenomenological model of decay by imposing simple and thermodynamically consistent physical requirements. A related physically consistent model can be found in the ‘warm’ inflationary scenario [10].

The first law of thermodynamics for open systems (with particle creation) and Einstein’s equations lead to a first-order equation for the expansion rate $H = \dot{a}/a$, whose source term is determined by the particle number N . A further equation in H and N arises from a simple model for the particle creation rate $\Gamma = \dot{N}/N$. We impose the thermodynamical non-equilibrium condition that Γ , and hence the entropy production rate, is maximal at the beginning of expansion. A further initial condition is that the initial vacuum for radiation is non-singular. We also require that $\Gamma > H$ initially, so that created particles are thermalized, while $\Gamma < H$ later on, as particle production becomes insignificant. Our simple model for Γ is naturally defined by the gravitational dynamics and conforms to these thermodynamical requirements. We decouple the equations to get a second-order evolution for H , and we find a remarkably simple exact solution $H(a)$.

Since the exit from inflation to the radiation era is smooth, we avoid the problem of matching at the transition. A similar smooth evolution has been used in [11, 12, 13], but in the context of adiabatic inflation, and without a consistent physical foundation. In effect, we show that the ad hoc form of $H(a)$ given in [11] follows from our simple physical conditions and thermodynamic arguments. In [14], a kinematic analysis is given for various non-adiabatic inflationary evolutions with smooth exit, but these evolutions are outside the scope of our model.

The choice of a as dynamical variable and the very simple form of $H(a)$ that meets the physical conditions, lead to elegant expressions for all parameters describing the radiation and decaying vacuum, and also to a physically transparent interpretation of these results, including the estimate of entropy. In addition, the equation for super-horizon scalar perturbations can be solved exactly for this form of $H(a)$.

We present in Sec. 2 the evolution equation for $H(a)$ and its simple solution, that

follow from our simple physical constraints. In Sec. 3 we analyze the thermodynamics of the radiation produced in the course of vacuum decay, and estimate the entropy of the created radiation. Sec. 4 contains a summary and concluding remarks. In a subsequent paper, we will discuss generalizations of the present model.

We use units with $8\pi G$, c and k_B equal 1.

2. The simple model

Consider a spatially flat Friedmann-Lemaitre-Robertson-Walker universe

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t) [dx^2 + dy^2 + dz^2] ,$$

containing matter with equation of state

$$p = \gamma\rho,$$

where γ ($0 \leq \gamma < 1$) is a constant parameter. Later we will specialize to the case of radiation, $\gamma = \frac{1}{3}$. The energy-momentum tensor of matter is

$$T_{\mu\nu}^M = \rho(t) [(\gamma + 1)u_\mu u_\nu + \gamma g_{\mu\nu}] , \quad u_\mu = \delta_\mu^0 ,$$

while the energy-momentum tensor corresponding to the quantum vacuum energy is

$$T_{\mu\nu}^Q \equiv \langle \hat{T}_{\mu\nu}^Q \rangle = \Lambda(t) g_{\mu\nu} .$$

Then the conservation equations $\nabla^\nu (T_{\mu\nu}^M + T_{\mu\nu}^Q) = 0$ reduce to

$$\dot{\rho} + 3(\gamma + 1)H\rho = -\dot{\Lambda} , \tag{1}$$

showing how energy is transferred from the decaying vacuum to matter. Note that (1) is equivalent to the energy balance of an imperfect fluid with scalar viscous pressure

$$\Pi = \frac{\dot{\Lambda}}{3H} .$$

This is an example of the known result that cosmological particle production may be interpreted as an effective bulk viscous pressure (see [15] and references cited there).

The field equations $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = T_{\mu\nu}^M + T_{\mu\nu}^Q$ are

$$3H^2 = \rho + \Lambda , \tag{2}$$

$$2\dot{H} + 3H^2 = -\gamma\rho + \Lambda , \tag{3}$$

and if both are satisfied then the energy equation (1) follows identically.

Following [11], we use a as a dynamical variable instead of t , and so we consider the Hubble rate as $H = H(a)$. Then equations (2) and (3) give

$$\rho(a) = -\frac{2}{(\gamma + 1)} aH(a)H'(a) , \tag{4}$$

$$\Lambda(a) = 3H^2(a) + \frac{2}{(\gamma + 1)} aH(a)H'(a) , \tag{5}$$

where primes denote d/da . Given $H(a)$ and γ , we can calculate $\rho(a)$ and $\Lambda(a)$. In order to determine further properties of $H(a)$, we will impose simple physical requirements.

We assume that the evolution of ρ is governed not only by expansion but also by the creation of particles, i.e. from the thermodynamic point of view, we have an open system. According to [3], the first law of thermodynamics generalized for open systems is

$$d(\rho V) + p dV - \left(\frac{\rho + p}{n} \right) d(nV) = 0, \quad (6)$$

where $p = \gamma\rho$ is the pressure, $n = N/V$ is the particle number density, N is the number of particles in the observable universe, and $V \propto a^3$ is the comoving volume of the observable universe. The thermodynamic equation (6) implies

$$\frac{(\rho a^3)'}{\rho a^3} + \gamma \frac{(a^3)'}{a^3} - (\gamma + 1) \frac{N'}{N} = 0,$$

which integrates to

$$\rho = \frac{A}{(\gamma + 1)} \left(\frac{N}{a^3} \right)^{\gamma+1}, \quad (7)$$

where A is a positive constant. From now on, we take $\gamma = \frac{1}{3}$, i.e. we assume that only radiation is created. This will also allow us to define entropy production via the photon number.

By equation (4), equation (7) can be rewritten as an *evolution equation for H , with source term determined by the particle number*:

$$\frac{d}{da} H^2(a) = -A \frac{N^{4/3}(a)}{a^5}. \quad (8)$$

This is the fundamental equation which follows from Einstein's equations and the thermodynamic equation (6). The creation mechanism is phenomenologically encoded in the source term $N(a)$. The model must be completed via an equation that determines N . If the creation rate of radiation $\Gamma = \dot{N}/N = aHN'/N$ is given, then equation (8) implies the second-order equation

$$3aHH'' + 3aH'^2 + (15H - 4\Gamma)H' = 0. \quad (9)$$

We seek a model in which most of the particle creation effectively takes place in the very early universe, starting from a regular vacuum. More precisely, we impose the following thermodynamical requirements:

- (a) Maximal entropy production rate (equivalently, maximal particle creation rate) at the beginning of expansion, so that the universe starts in a state furthest away from equilibrium and then tends toward equilibrium as the expansion proceeds.
- (b) A true (regular) vacuum for radiation initially, so that $\rho \rightarrow 0$ as $a \rightarrow 0$.
- (c) $\Gamma > H$ in the very early universe, so that we can treat the created radiation as

forming a thermalized heat bath. Subsequently, the creation rate should fall behind the expansion rate as particle creation becomes dynamically insignificant.

The fundamental physical quantities that are naturally defined by the gravitational dynamics in our model are the expansion rate H and the total energy density $U = \rho + \Lambda$. Both of these quantities can define in a natural way a gravitational creation rate Γ . The simplest model $\Gamma \propto H$ fails to satisfy requirement (c) above. Furthermore, it leads to a solution $H(a)$ of the evolution equation (9) which violates requirement (b). Therefore we propose $\Gamma \propto U$. This will satisfy requirement (a) if (b) holds, since the initial condition (b) implies that U approaches its maximum value $U(0) = \Lambda(0)$ as $a \rightarrow 0$. Below we show that the solution of equation (9) does verify condition (b). The increase of ρ due to creation in the very early universe partially offsets the decrease in Λ , so that U decreases slowly in the very early universe, and the entropy production rate remains high. Requirement (c) will be satisfied because in the radiation era, Λ becomes negligible and ρ decays like H^2 . Thus Γ decreases more rapidly than H , and will have fallen below H at some epoch.

Hence we propose the simple model that *the particle creation rate is proportional to the total energy density*. By the Friedmann equation (2), it follows that

$$\Gamma = \alpha H_e \left(\frac{H}{H_e} \right)^2, \quad (10)$$

where α is a dimensionless free parameter, and $H_e = H(a_e)$, where a_e is some fixed epoch. Equation (9) becomes the *decoupled, second-order evolution equation for $H(a)$* :

$$3aHH'' + 3aH'^2 + 15HH' - 4\frac{\alpha}{H_e}H^2H' = 0. \quad (11)$$

This equation has the first integral

$$3aHH' + 6H^2 - \frac{4\alpha}{3H_e}H^3 = 0, \quad (12)$$

where we have used the fact that H and aHH' tend to zero for large a (i.e. in the radiation era), in order to remove a constant of integration. The solution of equation (12) is

$$\beta \left(\frac{a}{a_e} \right) = \left[\frac{9}{2\alpha} \left(\frac{H_e}{H} \right) - 1 \right]^{1/2},$$

where β is a constant, and we have taken into account that H is a decreasing function. Evaluating at $a = a_e$, we see that

$$1 + \beta^2 = \frac{9}{2\alpha}, \quad (13)$$

which implies the constraint $\alpha \leq \frac{9}{2}$ on the creation parameter α . Re-arranging the solution, we find the remarkably simple form for the expansion rate that follows from

our thermodynamic model:

$$\frac{H}{H_e} = (1 + \beta^2) \left[\frac{a_e^2}{a_e^2 + \beta^2 a^2} \right]. \quad (14)$$

This solution approaches de Sitter inflation as $a \rightarrow 0$, i.e. $H \sim \text{constant}$, and it becomes radiation-like for $a \rightarrow \infty$, i.e. $H \sim a^{-2}$. It follows that as $a \rightarrow 0$, the cosmic proper time $t \rightarrow -\infty$. A naturally defined epoch is $a_{\text{ex}} = a(t_{\text{ex}})$ of exit from inflation, which is defined by $\ddot{a}(t_{\text{ex}}) = 0$, or equivalently $H(a_{\text{ex}}) = -a_{\text{ex}} H'(a_{\text{ex}})$. It follows from (14) that $a_e = \beta a_{\text{ex}}$.

These results reflect a subtle interplay between Einstein's gravitational dynamics and thermodynamical constraints on particle production. The decay of the vacuum into radiation drives inflation, but the same decay, by reducing the vacuum energy density, leads to a deceleration of expansion and a smooth exit from inflation. Since the initial vacuum for radiation is regular, the particle number and hence entropy are initially zero. As we show below, the total entropy produced in the observable universe in an infinite time is finite. This is consistent with the existence of a smooth exit, since unending inflation would produce infinite entropy. The initial rate of entropy production is maximal, reflecting the feature that the universe starts furthest from equilibrium and approaches asymptotically a state of equilibrium. We note also that the initial entropy production rate is a finite maximum value, rather than being unbounded from above. The latter possibility is ruled out by the de Sitter-like nature of the inflationary expansion, which implies that the expansion rate H is bounded from above (unlike power-law inflation for example).

The freedom in β , or equivalently α , by equation (13), provides us with an extra adjustable parameter. However, for simplicity, we will not use this freedom, since the subsequent results are not modified in any essential way for general β . Henceforth we take $\beta = 1$, i.e. $\alpha = \frac{9}{4}$, which means that $a_e = a_{\text{ex}}$. Thus, we arrive finally at the simple expansion rate

$$H(a) = 2H_e \left(\frac{a_e^2}{a_e^2 + a^2} \right). \quad (15)$$

This form of $H(a)$ was given in [11] as an ad hoc toy model to achieve smooth exit from inflation to radiation, but without a physical basis such as that given here. The expression for the cosmic proper time follows on integrating equation (15):

$$t = t_e + \frac{1}{4H_e} \left[\ln \left(\frac{a}{a_e} \right)^2 + \left(\frac{a}{a_e} \right)^2 - 1 \right].$$

3. Thermodynamics of radiation

On substituting equation (15) into equations (4) and (5), we find exact expressions for the energy density of radiation and the vacuum:

$$\rho(a) = 12H_e^2 \left(\frac{a}{a_e} \right)^2 \left(\frac{a_e^2}{a_e^2 + a^2} \right)^3, \quad \Lambda(a) = 12H_e^2 \left(\frac{a_e^2}{a_e^2 + a^2} \right)^3. \quad (16)$$

It follows that $\Lambda(0) = 12H_e^2$. Note that (15) implies $H(0) = 2H_e$. Note also that the effective bulk viscous pressure arising from particle production has the form

$$\Pi \equiv \frac{\dot{\Lambda}}{3H} = - \left(\frac{4\Gamma}{9H} \right) \rho = - \left(\frac{2a_e^2}{a_e^2 + a^2} \right) \rho.$$

Now ρ reaches a maximum at $a_m = a_e/\sqrt{2}$, with

$$\rho_m \equiv \rho(a_m) = \frac{16}{9} H_e^2, \quad \Lambda(a_m) = 2\rho_m.$$

Note also that ρ and Λ are equal at exit:

$$\rho(a_e) = \Lambda(a_e) = \frac{3}{2} H_e^2,$$

while for $a \gg a_e$, i.e. during radiation-domination,

$$\rho(a) \sim \frac{1}{a^4}, \quad \Lambda(a) \sim \frac{1}{a^6},$$

so that Λ rapidly becomes negligible.

The formulas (16) reflect the creation of radiation due to vacuum decay. The initial value $\rho(0) = 0$ confirms that the field corresponding to radiation is initially in a regular vacuum state. This means that the formula (16) gives an absolute measure of radiation produced in the universe, as a result of conversion of energy from the vacuum described by Λ .

Substituting equation (15) into equation (7) we get the exact form for the particle number

$$N(a) = N_\infty \left(\frac{a^2}{a_e^2 + a^2} \right)^{9/4}, \quad (17)$$

where N_∞ is usually taken to be about 10^{88} . It follows that about 2×10^{87} particles have been created at exit. Note that the initial conditions and the evolution equations in our model imply that *a finite number of particles is produced in the observable universe during the entire expansion.*

Since $N(0) = n(0) = 0$, the initial state of the field has no particles, i.e. it is a regular vacuum. The number density is

$$n(a) = 2^{9/4} n_e \left(\frac{a_e}{a} \right)^3 \left(\frac{a^2}{a_e^2 + a^2} \right)^{9/4}, \quad (18)$$

and reaches its maximum also at a_m .

The creation rate of radiation is given by equations (10) and (15) as

$$\Gamma = 9H_e \left(\frac{a_e^2}{a_e^2 + a^2} \right)^2 = \frac{9}{4} H_e \left(\frac{H}{H_e} \right)^2. \quad (19)$$

In order to define the radiation temperature (and then entropy), we would like to invoke the standard black-body relation. A justification of this is as follows. From equation (19), it follows that the creation rate $\Gamma(a)$ exceeds the expansion rate $H(a)$ for

$$a < \sqrt{\frac{7}{2}} a_e.$$

Thus it is reasonable to treat the created radiation as forming a thermalized heat bath in the initial stage of expansion. For $a > \sqrt{\frac{7}{2}} a_e$, the creation rate falls behind the expansion rate, and created particles will be out of equilibrium. However, by this stage of the expansion, the energy density in newly created particles is too small to disturb the effective thermalization. Thus it seems reasonable to use the black-body relation for the radiation throughout the expansion, and to define the temperature by

$$T(a) = \frac{1}{3} \frac{\rho(a)}{n(a)} = \frac{H_e^2}{2^{1/4} n_e} \left(\frac{a_e}{a} \right) \left(\frac{a^2}{a_e^2 + a^2} \right)^{3/4}, \quad (20)$$

where we have used equations (16) and (18). At the initial radiation vacuum, it is clear that $T(0) = 0$, and T increases to its maximum value at a_m :

$$T_m \equiv T(a_m) = \left(\frac{2}{27} \right)^{1/4} \frac{H_e^2}{n_e}.$$

This temperature may be thought of as analogous to the reheating temperature in standard models. During the radiation era, i.e. for $a \gg a_e$,

$$T \sim a^{-1},$$

in agreement with the standard result for free radiation in an expanding universe.

The formulas for ρ and n can be presented in the thermodynamic form

$$\rho = 24 \left(\frac{n_e^4}{H_e^6} \right) T^4, \quad n = 8 \left(\frac{n_e^4}{H_e^6} \right) T^3. \quad (21)$$

Combining now the Gibbs equation

$$TdS = d(\rho V) + pdV,$$

with equation (6), and using the definition (20) of T , we obtain the entropy of radiation in the observable universe as

$$S(a) = 4N(a), \quad (22)$$

so that $S(0) = 0$ as expected. This gives a reasonable value for the entropy produced during the overall evolution of the universe:

$$S_\infty = 4N_\infty \approx 4 \times 10^{88}. \quad (23)$$

In the standard model (adiabatic inflation followed by reheating), one can estimate the entropy production due to reheating by matching exact de Sitter inflation to an exact radiation era, with an instantaneous transition at a_e . This gives [11]:

$$S_e = \frac{4}{3}g^{1/4}\rho_e^{3/4} = S_\infty,$$

where $g \sim 100$, and leads to a value of the same order of magnitude as our result.

4. Conclusion

We have considered a simple and thermodynamically consistent scenario encompassing the decay of the vacuum, the creation of radiation and entropy, and a natural smooth transition from inflationary to radiation-dominated expansion. In order to treat all matter in the universe as created from a regular vacuum, we impose the condition that $\rho \rightarrow 0$ as $a \rightarrow 0$. We impose the further initial condition that the entropy production rate is a maximum. A simple model for the particle creation rate Γ , i.e. that Γ is proportional to the comoving total energy density, is shown to be consistent with these initial conditions and with the requirement Γ should start above and then fall below the expansion rate H . We showed that the field equations and the first law of thermodynamics (6) generalized for open systems with creation of matter, then imply a second-order evolution equation (11) for $H(a)$. This equation has the remarkably simple solution (14), and we used this together with black-body thermodynamics and the Gibbs equation to define the temperature and entropy.

Our postulate $\Gamma \propto H^2$ for the creation rate, can be contrasted with other work. For example, in [3], $\dot{N} \propto H^2$, while Γ is constant; in [8], $\Lambda \propto \exp(-t/\tau)$, and there is no simple expression for Γ in terms of H . The postulates in these papers are ad hoc, whereas we have tried to give a thermodynamic justification for our postulate. It is also interesting to compare our decaying-vacuum/ radiation model with scalar-field/ radiation models [10, 16]. In these latter models, a phenomenological term is introduced into the Klein-Gordon equation for the scalar field ϕ , describing the interaction between the decaying field and radiation. The resulting energy balance equation is

$$\dot{\rho} + 4H\rho = \Gamma_\phi \dot{\phi}^2,$$

where Γ_ϕ is the phenomenological decay rate. Comparing this with our energy balance (1), and using (7), we see that $\Gamma_\phi \dot{\phi}^2$ corresponds to our $\frac{4}{3}\Gamma\rho$. It is not obvious whether a scalar field interpretation of our model exists that would produce the rate $\Gamma_\phi \propto H^2$. This is a subject for further investigation.

An important feature of the Hubble rate (15) is that $\lim_{a \rightarrow 0} H(a) = \text{constant}$ and $\lim_{a \rightarrow 0} H'(a) = 0$. According to equations (16) and (17), this avoids any divergences in ρ and N at $a = 0$. In addition, it follows from the equations (16), (17), (20) and (22) that all thermodynamic parameters describing the created radiation vanish initially, i.e. all forms of matter which we observe now in the universe were in a state of regular vacuum, and all constituents of the universe were created from this vacuum in the course of the decay of the vacuum energy density Λ from its initial value $12H_e^2$. We also showed that our requirements imply finite particle and entropy production during the entire expansion of the observable universe.

Our simple model leads to exact expressions for $t(a)$ and for the thermodynamic variables. In addition, the form (15) of $H(a)$ leads to an exact solution for the large-scale modes of scalar perturbations, described gauge-invariantly by Bardeen's potential Φ . The solution is [17]:

$$\Phi = C_+ \left(\frac{a^2}{a_e^2 + a^2} \right) + C_- \left(\frac{a_e}{a} \right) \left(\frac{a_e^2}{a_e^2 + a^2} \right),$$

where $C'_\pm = 0$. We will not discuss the important question of the source of these perturbations, except to mention that, as pointed out in [10], non-adiabatic inflationary scenarios allow for the possibility that the seeds of density perturbations are of predominantly thermal, rather than quantum, origin.

Further discussion of the physical processes taking place during non-adiabatic inflation, and of their effects on baryogenesis, the microwave background radiation and structure formation, is given in [10, 14, 7, 6, 18].

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